The Development of Student Cohorts for the Enhancement of mathematical Literacy in Under Served populations

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PEAr Deliverable

Introduction

What is a PEAr?

ETS has developed Product Efficacy Arguments (PEAr) for a variety of assessments, interventions or products/services. The PEAr consists of a logic model along with a targeted literature review for each specific claim in the logic model. Creating the PEAr is one mechanism to document a formal theory of action for the Algebra Project (AP) which would a) give the program a better understanding of the claims of AP and their foundation in research and b) create a strong foundation for future research proposals for further evaluation, research, and development of AP.

What is the process that was followed?

ETS staff worked with AP staff to develop a logic model diagram that conveyed the various intermediate outcomes for teachers and students in service of the ultimate improved student learning outcome. We reviewed existing AP research and evaluations, as well as empirical research studies conducted in other contexts. A summary of key findings for each claim was developed. In addition a description was written of how the AP structures and supports are intended to lead to the identified changes in the theory of action. Each citation in the full research summary was independently reviewed by ETS Research staff to ensure both that the original research was of sufficient rigor and that the summary accurately reflected the relevant findings for the PEAr. Finally, experts in the fields of validity, mathematics education, and intra-personal skills reviewed the full manuscript to comment on the plausibility of the theory of action and the strength of the supporting research. Final revisions were made to the manuscript in the light of these comments. The findings from the complete process are summarized below and the full manuscript follows.

What are the findings?

In the PEAr there are 13 claims. With respect to the strength of support provided the PEAr, reviewers felt that nine of the 13 claims have a reasonable level of empirical support. This indicates that other researchers have asked similar questions about changing teacher practice, student behaviors and academic achievement, and found positive results. However, reviewers felt that there were four claims where more empirical research may be warranted.

Specifically, reviewers felt that additional research support linking changes in teacher practice to the initial student outcomes in this theory of action may be needed (claims 6, 7 and 9). While there is research related to the development of academic perseverance and academic mindset, there are limitations to its applicability to this specific theory of action:

- constructs are not consistently defined across programs;
- assessments of these constructs do not always have sufficient reliability and validity evidence;
- in many cases the purpose of the research was to connect changes to practice to student achievement or changes in students to student achievement, rather than connecting teacher behavior to student behavior.

Finally, claim 13 connects student outcomes to graduation in four years, ready for college math for college credit, and ready to meet math literacy requirements for any other career choice. Given the specificity of this final outcome it was difficult to locate literature to support the claim.

How should the findings be used?

The PEAr process allows the AP to explicate a theory of action for an intervention, identify existing literature in support of that theory, and identify gaps in the literature. Overall, reviewers of the AP PEAr felt that the theory of action was logical and plausible, but that more empirical work was needed, particularly around the areas of academic mindset and academic perseverance. Lack of identified literature does not indicate weakness in the theory of action, but rather should be used to guide the development of a robust research agenda into individual questions, and when appropriate can guide a full program evaluation. While ETS partnered with the Algebra Project in the development of this PEAr, the final manuscript does not constitute an ETS endorsement of the Algebra Project program.

Product Efficacy Argument for the Algebra Project

The development of a product efficacy argument (PEAr) is an important step in the development and evaluation of an educational intervention. A PEAr helps product developers make informed decisions about the structure and scope of the product, evaluators make decisions about what to measure, and educators and clients make informed decisions about the product's use.

A PEAr contains a description of the product and the underlying theory of action which indicates how a product is intended to work when implemented appropriately. The theory of action is illustrated through a diagram that connects the product to both student and teacher outcomes, as appropriate. The theory of action is then followed by summaries of research that support the theory. To this end, the research summaries present evidence from studies that did not involve the intervention, but that generally support this intervention's theory of action.

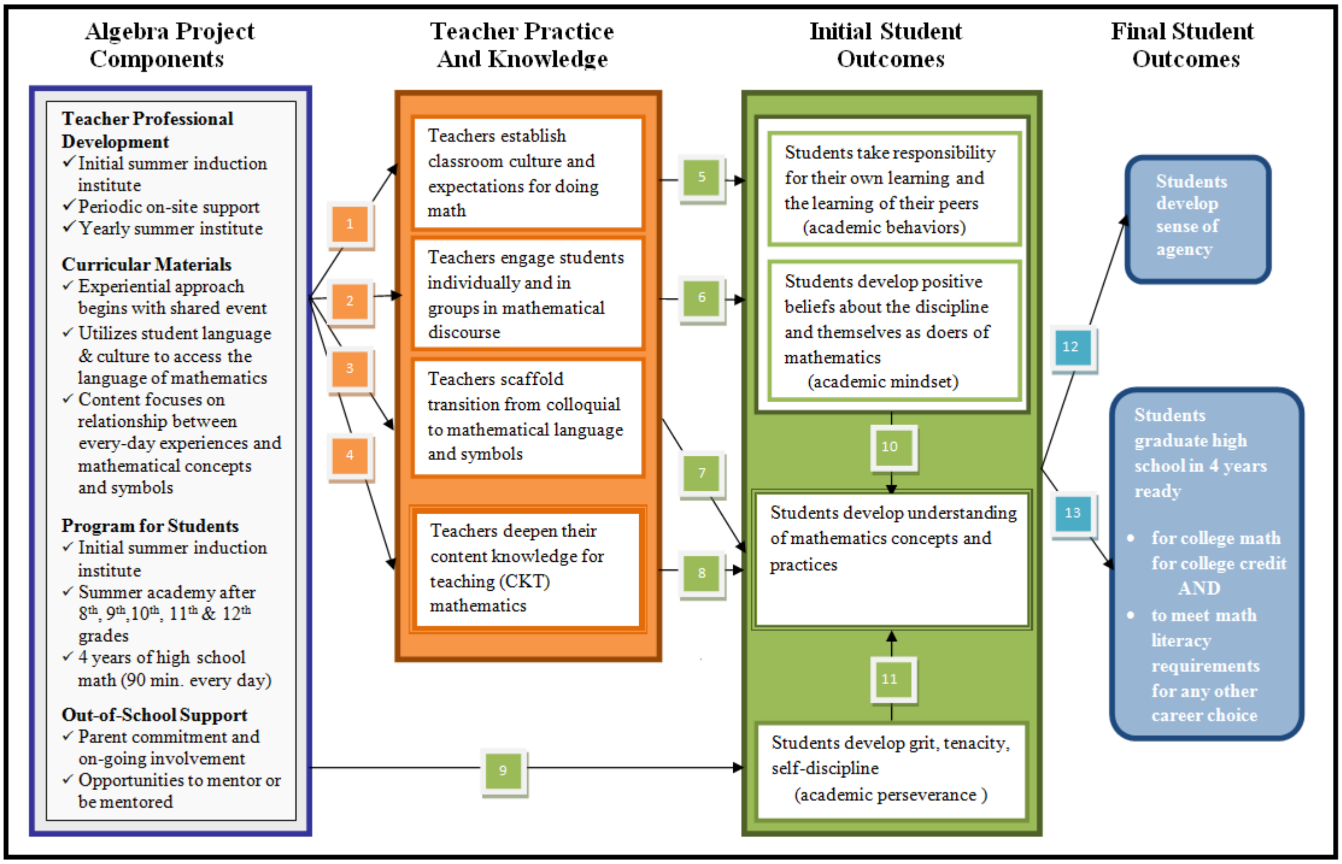
Product Description

The Algebra Project is a mathematics literacy effort designed to help the lowest-achieving students acquire the mathematical knowledge and practices necessary to: (a) succeed in college and career and (b) participate fully as informed citizens in a highly technical, global society. The central premise is that access to mathematical literacy is afforded when students (individually and in groups) are engaged in discourse and reasoning about everyday experiences, the relationships between those experiences, and the mathematical concepts and symbols that derive from them. Since its inception in 1982, the Algebra Project has focused on what to teach and how to teach it in ways that motivate students to expend the effort to do and learn mathematics despite personal challenges and institutional obstacles. In school, curricular materials and an ongoing program of teacher professional development promote learner-centered instruction and a classroom culture that: (a) fosters individual and shared responsibility for learning, (b) establishes high expectations for student performance and work habits, and (c) supports student discourse and efforts to communicate their thinking and reasoning. Out of school, additional optional support is provided through the Young Peoples Project (YPP) Math Literacy Workers (MLWs), who provide peer-to-peer mentoring to develop students' mathematical competence, self-efficacy and confidence. Coalitions of community-based stakeholders work together with parents and caregivers during school/district regime changes to ensure long-term sustainability of the program.

The Theory of Action

The diagram below displays the theory of action for the Algebra Project. The diagram begins with a list of the intervention's components. A series of numbered arrows then connects the product to intermediate outcomes and final outcomes.

Each arrow represents a specific hypothesis for what is expected to happen when the product is implemented. A summary of salient, relevant research for each hypothesis is detailed in the following sections. The research evidence presented is from studies that did not use the intervention, but that support the theory of action. The arrows and research summaries are numbered and color-coded for easy identification. Orange claims and outcomes refer to changes in teacher practice and knowledge; green claims and outcomes refer to intermediate student outcomes; and blue claims and outcomes refer to the final student outcomes.



Supporting Research

For each hypothesis three pieces of information are presented: (1) the specific claim or hypothesis identified in the theory of action (2) research that supports the hypothesis and details how the product may lead to the identified outcome; and (3) how the product addresses the research.

Claim 1-3

When mathematics teachers are provided with coherent, sustained professional development around learner-centered curriculum and instruction, they may expand their repertoire of responsive teaching practices.

Research has shown that ongoing, collaborative professional development can have a sustained impact on teaching practices (Butler, Lauscher, Jarvis-Selinger, & Beckingham, 2004; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Franke, Carpenter, Levi & Fennema, 2001, Garet, Porter, Desimone, Birman, & Yoon, 2001).

Fennema et al. (1996) examined changes in teachers' beliefs and instructional practices over a four-year period in which the teachers participated in a Cognitively Guided Instruction (CGI) professional development program focused on increasing teacher's understanding of students' mathematical thinking. Twenty-one 1st through 3rd grade teachers in three schools participated in a three-year CGI professional development program focused on helping teachers to construct models of the development of children's mathematical thinking based on extant research in domains of elementary mathematics (e.g., addition and subtraction). Teachers were provided with in-school support (workshops, meetings, mentoring) during the first two years and this support was reduced during Year 3. During follow-up sessions, teachers discussed their students' thinking and learning in relation to the CGI framework and discussed instructional strategies to elicit student thinking in mathematics (e.g., engage in responsive teaching). Teachers were observed and interviewed prior to the intervention and at the end of each program year. Data were coded for level of teacher engagement with children's mathematical thinking. Each level had a set of descriptions of teacher beliefs and practices (e.g., at the lowest level, does not ask the children how they solved the problem; at the highest level, creates opportunities to build on children's mathematical thinking). According to analyses of three years of data on teachers' consideration of students' mathematical thinking, over 90% of the teachers in the sample demonstrated increases in their consideration of students' mathematical thinking over time. Results also suggested that changes in instructional practices were reflected in increased student achievement. A follow-up study (Franke et al. (2001), in which teachers in the initial sample were observed and interviewed four years after the professional development ended, found that a slight majority of the teachers were still at the highest levels of engagement four years later. Teachers who had exhibited lower levels of engagement in year one tended to remain at those levels also.

Garet et al. (2001) examined the characteristics of professional development associated with teachers' self-reported change in knowledge, skills and instructional practices. Teachers were not directly observed. A nationally representative sample of teachers who attended mathematics and science activities supported at least in part by funds provided by the federal Eisenhower Professional Development Program were surveyed. Data on the characteristics of the professional development were collected from a teacher activity survey in which teachers were asked to describe the characteristics of the professional development activity they attended and the extent to which their instructional practices, as well as their knowledge and skills had been enhanced. To assess the effects of professional development participation on knowledge and skills, teachers were asked to indicate to what extent (1 = *not at all* to 5 = *to a great extent*) their knowledge and skills had been enhanced in six areas: curriculum, instructional methods, approaches to assessment, use of technology, strategies for teaching diverse student populations and deepening knowledge of mathematics. To assess the effects of professional development participation on instructional practices, teachers were asked to what extent (0 = *no change* to 3 = *significant change*) they made changes, as the result of the professional development, in six areas of teaching practice: content, cognitive challenge of activities, instructional methods, assessments used, technology use, and approaches to student diversity. Teachers' responses were averaged across the six items to create two scales, one measuring enhanced knowledge and skills and one measuring change in teaching practice. Teachers who report enhanced knowledge and skills as a result of their professional development experience are also likely to report changing their practice ($\beta = 0.44$, $p < .001$). Furthermore, the coherence of professional development activities has a positive effect on change in teaching practice ($\beta = 0.21$, $p < .001$) over and above the effects of knowledge and skills.

The Algebra Project supports the development of responsive teaching practices through a sustained program of professional development that includes a three-week summer induction program, in-class support throughout each of four school years, and yearly summer institutes for three summers. During the initial summer induction program teachers are introduced to the Algebra Project pedagogical approach, the curriculum, and strategies for establishing a positive learning environment where students can do mathematics. Opportunities are provided for the teacher to begin to work with his/her students with support from Algebra Project staff. Periodic on-site support over the school year is used to review and reinforce the mathematical concepts and practices introduced during the summer in ways that are responsive to teacher needs as well as those of students.

Claim 4

When teachers are provided with coherent, sustained professional development around student thinking and learning in mathematics they may expand their content knowledge for teaching.

Research has shown that sustained professional development that focuses on how students learn and provides opportunities for active learning, may increase teacher content knowledge for teaching mathematics.

Hill and Ball (2004) quantified the effect of professional development on the improvement of content knowledge for teaching using an assessment instrument designed for this purpose. The Mathematical Knowledge for Teaching (MKT) assessment includes items that tap into specialized knowledge for teaching mathematics, such as interpreting student work, explaining mathematical procedures, comparing explanations of mathematical concepts, choosing or comparing representations, and appraising unfamiliar solutions. Analyzing pre- and post-test data from 398 elementary school teachers in 15 professional development summer institutes, researchers found an average pre-to-post gain of between one-third and one-half of one standard deviation, which was statistically significant ($p < .0001$). There was, however, wide variation among institutes in the magnitude of the change in MKT. Approximately one sixth of institutes had teachers who gained, on average, a standard deviation or more. Because the MKT assessment was not designed for a particular professional development program, the authors surveyed participants to gather information on features of the summer institutes including length of each institute and content. Length of the institute (measured in days) was a significant predictor of the institute's effectiveness, as was opportunity to engage in mathematical analysis, reasoning, and communication.

Bell, Wilson, Higgins, and McCoach (2010) examined the impact of the Developing Mathematical Ideas (DMI) professional development program on teacher's mathematical knowledge for teaching (MKT). As part of DMI, K-8 teachers work with a trained facilitator to learn about specific mathematics topics, children's ideas about this area of mathematics, and instructional approaches that engage and support student reasoning. Teachers volunteered to participate in DMI (treatment) or serve in the non-DMI comparison group (control). Teachers were not randomly assigned nor matched on demographic variables although teachers in the two groups taught in the same districts. The final sample included 234 teachers across nine sites. All teachers completed pre- and post-test measures of MKT. Post-test measures were administered to treatment teachers after completing 2 DMI modules. While results should be interpreted with caution since the experimental group opted-in, the findings indicate that teachers who participated in DMI showed greater gains than the control teachers on both multiple choice (0.36 standard deviation units) and open-ended questions (0.70 standard deviation units), but the magnitude of the difference was larger for open-ended questions (3.4 points versus 1 point).

The Algebra Project

The Algebra Project provides professional development (PD) on its instructional units during both summers and the academic year. The focus of professional development is to engage teachers on (1) deepening their mathematical content knowledge and analysis, (2) strengthening their classroom pedagogy and mathematical reasoning, and (3) developing their ability to build a culture of communication, discussion and collaboration in the mathematics classroom. During teachers' first summer with the Algebra Project they attend a two week summer institute which has a focus on the mathematical content, reasoning, and analysis they will be developing with 9th grade students across the school year. This content institute is then followed by a three-week induction academy where Algebra Project PD specialists and teachers work with their rising 9th graders on a few experiential instructional units that begin to establish the mathematics and the foundation for the classroom culture and communication that they will continue to build across the school year. This constitutes a total of 125 hours of PD for teachers during the first summer. In each of the following three years teachers work through a summer unit consisting of a 5-day content institute and a two-week student academy consisting of their rising 10th, 11th and 12th graders respectively, where they engage on mathematical content, analysis, reasoning, and communication. This constitutes a total of 75 hours of professional development per summer for three summers. During the school year teachers are provided with classroom support and coaching on a monthly basis (5 days per month), with the same focus on strengthening mathematical content knowledge, analysis, pedagogy, reasoning, and classroom mathematical communication, and culture.

Claim 5

When teachers establish a positive classroom culture and expectations for doing math, students are more likely to assume responsibility for their own learning and the learning of their peers (academic behaviors).

Research has shown that a positive classroom culture (e.g., mutual respect among students and the teacher, kindness, collaboration, opportunities to participate, helpfulness, etc.) and teacher expectations that all students can do mathematics is linked to increased levels of student involvement, responsibility, and engagement in learning.

In a study examining the relationship between teacher motivational support and student engagement, Turner, Christensen, Kackar-Cam, Trucano, and Fulmer (2014) found "that teachers who were observed to use more strategies that supported motivation had students who were observed to be more engaged" (p. 15). Eight sixth- through eighth-grade teachers from a rural Indiana public school took part in a three-year intervention that included professional development that focused on improving student engagement through the use of instructional strategies rooted in four motivation constructs (belongingness, competence, autonomy, and meaningfulness). Complete data sets were obtained from six of the eight teachers. An observation instrument was used to measure a variety of student-teacher interactions across six categories for both student engagement and motivational support. Reviews of trends in teachers' scores on a motivational support scale ($\alpha = .95$) at 12 time points over three years (4 per year) indicated that teachers fell into one of two categories over the course of the intervention: an upward trend in motivational support ($n=3$) or a flat (stable) trend ($n=3$). Teachers were thus divided into upward and stable groups for the outcome analyses. Paired-sample *t* tests revealed that, among teachers in the upward group, motivational support increased significantly from Year 1 to Year 2 ($t(2) = -15.69$, $p = .004$) and between Year 1 and Year 3 ($t(2) = -5.00$, $p = .04$). In addition, student engagement significantly increased from Year 1 to Year 2 among the upward group teachers ($t(2) = -5.77$, $p = .03$). However, among the stable group, there was no statistically significant change in motivational support or student engagement in Years 1, 2, or 3. Thus, on average, teachers in the upward group progressively implemented more strategies that supported student engagement from year to year. Independent-sample *t* tests were also conducted to compare teachers in the upward group to those in the stable group on both motivational support and student engagement for each year. There were no differences between the groups in Year 1, suggesting that teachers were using similar motivational strategies at the beginning of the intervention. However, both motivational support and student engagement were significantly higher among the upward group, compared to the stable group teachers, in Year 2: motivational support, $t(4) = -3.14$, $p = .04$; student engagement, $t(4) = -3.37$, $p = .03$; and Year 3: motivational support, $t(4) = -6.43$, $p = .003$; student engagement, $t(4) = -4.63$, $p = .01$. These results suggest that teachers who were observed to use more strategies that supported motivation had students who were observed to be more engaged.

Boaler (2008) analyzed data collected as part of a study examining different mathematics teaching approaches. In this paper, the data was analyzed through the lens of *relational equity*. She defines relational equity as equitable relations in classrooms that include mutual respect and shared responsibility. The four-year longitudinal study of three high schools in California involved roughly 700 students and the observation of over 600 hundred hours of instruction. A blend of qualitative and quantitative methods was used to analyze a wide range of data sources – observations, student interviews, questionnaires, video-taped problem-solving sessions, and assessments. Looking across data sources from the three schools the researchers classified each school as having either a traditional approach to mathematics or a reform-oriented approach. One of the three schools was classified as a having a reform oriented approach. This school served a diverse student body from low-income homes and placed all students in heterogeneous algebra classes as they entered high school. Instruction in these classes emphasized group work (72%) with more teacher questioning and less teacher lecturing (4%) and more time set aside for students to present their work (9%) (p.10). The other two schools were classified as having a traditional approach to mathematics instruction. These schools served a less diverse, more suburban student population and tracked students into classes by ability levels. Classes in these two schools utilized traditional models of instruction such as teacher lecturing (21%), individual seat work (48%), and little time set aside for students to present their work (0.2%) (p.9). The authors compared the commitment to the learning of others at the reform high school to the traditional high schools and identified two themes: (1) reciprocity which refers to "the concern students developed for each other's learning" and (2) responsibility when things go wrong which refers to "the actions [students] took when other students were not working". The researchers found that 59% of the students in the reform school "communicated high levels of responsibility towards their peers compared to 5% of the students taught traditionally, a statistically significant difference ($X^2 = 2.3125$ $p<0.001$, df=2)". The researchers concluded that overall students at the reform school viewed learning and mathematics as a "collective rather than an individual endeavor" due to the intentional work that teachers did to explicitly teach students to be responsible for their peer's learning.

In a study examining the links between classroom climate, instructional quality, and student behavior, Matsumura, Slater, and Crosson (2008) found that student-to-student interactions were positively impacted by teacher efforts to create learning environments rooted in respect and collaboration. Thirty-four sixth- and seventh- grade middle school teachers from five urban east coast schools serving mostly low-income minority students (n = 608) were observed facilitating classroom discussions for two consecutive days. Thirteen of the 34 teachers taught mathematics and were observed facilitating a discussion specifically based on a problem-solving activity (26 lessons in total). A number of rubrics were used to measure the quality of a variety of behaviors and interactions, including but not limited to: level of respect teachers exhibit towards students, opportunities for purposeful collaborative work, the presence of established prosocial and positive rules for respectful peer-to-peer behavior, and student-to-student helpfulness with assignments, participation in classroom discussion, and students making connections to their peers' responses. Results showed a positive association between the level of respect teachers showed their students and the degree of respect students displayed towards each other ($r = .64$, $p < .001$). Additionally, multiple linear regression analysis was used to determine if teacher behaviors predicted student behavior. Results showed that "the degree of respect teachers showed students significantly predicted students' behavior toward one another" (p. 306) ($B = .546$, $t_{11} = 3.316$, $p = .002$). Finally, greater numbers of students participated in class discussions when clear rules for prosocial behavior and mutual respect were presented and upheld ($B = .330$, $t_{11} = 3.347$, $p = .002$).

The Algebra Project devotes significant time and attention supporting teachers to establish a learning environment where the norm is one-on-one and small group work rather than whole class discussion. In this way, teachers can attend to the needs of individual students. To allow for this one-on-one student-teacher interaction, all other students must take responsibility for doing their work and helping their peers in small groups.

Claim 6

When teachers engage students in mathematical discourse, students are more likely to develop positive beliefs about the discipline and themselves as doers of mathematics (academic mindset).

There is limited empirical research in the field that connects the type of mathematical discourse promoted by the Algebra Project to students' academic mindset. However, research has investigated the impact of discourse in mathematics classrooms specifically related to oral feedback and modeling as well as scaffolding within instructions. While not a direct parallel to the Algebra Project, these results provide some support for the relationship between discourse type and students' academic mindset.

Siegle and McCoach (2007) investigated the impact of mathematics instruction that incorporates self-efficacy teaching strategies on development of students' mathematics self-efficacy. The teaching strategies focused on three areas: goal setting (e.g., facilitating class reflection on lesson accomplishments), teacher feedback (e.g., complimenting students on specific skills), and modeling (e.g., facilitating student demonstration of a skill during the lesson). Using a cluster-randomized pre-test/post-test design, 40 fifth-grade classrooms (872 students) from 15 schools were randomly assigned to either the treatment or control group. Teachers in the treatment group received training on self-efficacy teaching strategies. Following the training, teachers in the treatment group implemented the strategies with students during a 4-week mathematics instructional unit on measurement. Teachers in the control group taught the same 4-week mathematics unit but did not receive the training nor implement new strategies. A self-report questionnaire was developed to assess students' self-efficacy related to their ability in measurement. Students completed the questionnaire prior to ($\alpha = .96$) and following ($\alpha = .97$) completion of the instructional unit. Multilevel regression analysis showed that after controlling for gender, mathematics ability and pre self-efficacy, the average post self-efficacy was 0.52 points higher for students in the treatment group compared to students in the control group, with an effect size of 0.46. These results indicate that when accompanied by teacher feedback on specific skills, classroom discourse that facilitates reflection on lesson accomplishments and student modeling of successful performance results in improvement of students' self-efficacy in math.

Turner, Meyer, Cox, Logan, DiCintio, and Thomas (1998) studied the relationship between teachers' scaffolding of instruction and students' quality of experience in elementary mathematics classrooms. The classroom discourse of 7 fifth- and sixth-grade teachers was recorded for 4 to 5 mathematics lessons, and 6 student participants from each class completed a response log after each observed lesson, for a total of 42 student participants and 181 log responses. The log asked students to report on their perception of their affect during instruction. In addition, students answered two questions about the level of challenge and their mathematics skills during the specific observed lesson. A combination of qualitative and quantitative analysis was adopted in the study. Students who reported above-average challenge and skills were also significantly more likely to report positive feelings: i.e., that they were more involved ($M = 0.22$, $t[1,77] = 2.36$, $p = 0.021$); open ($M = 0.19$, $t[1,77] = 1.97$, $p = 0.05$); relaxed ($M = 0.18$, $t[1,77] = 2.07$, $p = 0.042$), and intrinsically motivated ($M = 0.36$, $t[1,77] = 2.80$, $p = 0.007$). An analysis of variance (ANOVA) and planned comparisons of student responses to the questions about challenge and skill revealed that students reported above-average challenge and skills in three of seven teachers' classrooms. Further, discourse analysis of transcripts from the observed lessons revealed that these three teachers provided not only cognitive, but also motivational and emotional supports, by negotiating meaningful learning, promoting student control of thought and actions through transfer of responsibility, and providing intrinsic supports for learning. In contrast, the other four teachers retained ownership of mathematics knowledge and motivated students through extrinsic rewards and praise, or threats for noncompliance. In summary, this study shows that students are more likely to have a positive mathematics classroom experience when teacher discourse provides the cognitive, motivational, and emotional supports to help students meet challenging goals.

The Algebra Project emphasizes the importance of engaging students in discussions of mathematics, initially through shared learning experiences which then transition to using more formalized mathematical representations. In so doing, students experience success in mathematics and shift mindsets from not liking or wanting to do mathematics to seeing it as something that they are able to engage in.

Claim 7

When teachers use responsive discourse strategies to scaffold the transition from colloquial to mathematical language and symbols, students are more likely to develop an understanding of mathematics concepts and practices.

Research has shown that instructional practices are effective in supporting the development of student knowledge of mathematics when they attend to the way language is used and apply strategies for moving students from informal everyday ways of thinking about mathematics to more technical and precise meanings. Moreover, applying responsive discourse techniques, and engaging students in mathematical discussion with one another and with the teacher can lead to improved student learning (Boaler & Staples, 2008).

Boaler and Staples (2008) conducted a comparative study of two instructional approaches in three urban high schools in California (traditional and reform-oriented). Results lend additional support to the idea that a "communicative" approach to mathematics teaching has an impact on student learning. In an urban high school in California, students in the reform-oriented program worked on algebra and geometry tasks in heterogeneous classes, where students would frequently be asked to explain their work to each other. A review of classroom observations showed differences in teacher use of lectures (4% and 21% for reform-oriented and traditional teachers, respectively), time students spent working on problems (5.7 min/problem in reform versus 2.5 min./problem in traditional), and teachers' use of a variety of question types (62% procedural, 17% conceptual, 15% probing and 6% other in reform-oriented classrooms). Teachers in the two traditional classes averaged 98% procedural questions. At the beginning of the year, the reform classes started with lower average mathematics achievement scores ($M = 16.0$) than the students in the traditional classes ($M = 22.23$) ($t = -9.141$, $p < .001$, $n = 658$). By the end of the second year, the students in reform classrooms ($M = 26.47$) were outperforming students in traditional classrooms ($M = 18.34$) on algebra and geometry tests ($t = -8.309$, $p < .001$, $n = 512$). Additionally, the reform classes successfully reduced the achievement gap between groups of students belonging to different ethnic groups.

The Algebra Project has developed mathematics curricula that leverage students' everyday language to access the language and symbols of mathematics. Through individual and group work, students document and discuss a shared event (e.g., a subway ride) initially using everyday language and pictorial representations and shifting to conceptual and symbolic ones over time, eventually transitioning to the structured language of mathematics and associated symbolic representations. In this context, responsive teaching practices are critical for improved mathematical discourse and reasoning.

Claim 8

When teachers increase their mathematical knowledge for teaching, students are more likely to develop an increased understanding of mathematics concepts.

Research has shown promising links between mathematical knowledge for teaching and student achievement. Hill et al (2005) conducted a study to explore whether and how teachers' mathematical knowledge for teaching contributes to gains in students' mathematics achievement. Teacher surveys including measures of content knowledge for teaching mathematics (CKT-M) and student standardized test scores were collected for a sample of 115 elementary schools, with high poverty schools being overrepresented, over a two-year period. The authors used a linear mixed-model methodology in which first and third graders' mathematical achievement gains over a year were nested within teachers, nested within schools. Results indicated that teachers' CKT-M was a significant predictor of student gains. Students at both grades gained approximately 2.25 points on the Terra Nova for every standard deviation difference in teachers' CKT-M. The effect of CKT-M was larger than the effect of socioeconomic status (SES) at 3rd grade (2.13) but less than the effect of SES at 1st grade (3.96).

Baumert et al (2010) investigated the extent to which teachers' content knowledge (CK) and content knowledge for teaching mathematics (CKT-M), which includes pedagogical content knowledge (PCK), contribute to high quality instruction and student achievement. Findings from a one-year study conducted in Germany with a nationally representative sample of 10th grade mathematics teachers and their students showed a positive effect of CKT-M on students' learning gains, with 39% of the variance in achievement between classes explained by differences in PCK. Across the entire sample, "completely comparable classes taught by teachers with PCK scores in the lower or upper quintile of the competence distribution can thus be expected—all things being equal—to show learning gains in the range of about *d* less than or equal to 0.15 and *d* greater than or equal to 0.55, for those with low and high CTK-M, respectively" (Baumert et al., 2010, p. 165).

The Algebra Project works with teachers to support the development of content knowledge for teaching in two ways. First, the curricular units are experienced-based and designed to help students relate everyday experiences to fundamental concepts in mathematics. This structure requires teachers to understand the connections, how to scaffold learning so students make the connections, and the ways in which students understand or misunderstand the mathematical concepts that underlie the experience. Second, summer institutes and in class support focus on how to facilitate thinking and reasoning and how to respond to students (individually and in groups) in ways that promote thinking, reasoning, and advance learning.

Claim 9

When students are provided extended learning opportunities and support, students are more likely to persevere in their efforts to learn and do mathematics (academic perseverance).

The Algebra Project model assumes extended learning opportunities, mentoring, support, and opportunities to mentor others will lead to increases in student perseverance (e.g., grit, tenacity, and self-control) for learning and doing mathematics. This view is supported in the literature (Pierce & Shields, 1998; Major, 2013); however, empirical studies have focused primarily on measuring the relationship between perseverance and learning outcomes rather than on the malleability of perseverance and the impact of opportunities on perseverance.

While we did not locate studies in the context of mathematics, interventions that include the opportunity to mentor or be mentored have been shown to increase self control, grit, and perseverance.

A quasi-experimental evaluation of the Be A Star out-of-school-time program found that treatment students made significantly greater gains on a measure of self-control (Pierce & Shields, 1998). Thirty-eight after school groups (ages 9-12) across five locations of a community-based organization participated in the study across 3 program years (1992–1993, 1993–1994, and 1994–1995). At the beginning of the study, 17 groups (n = 386) were assigned to the treatment condition and received the Be A Star curriculum, while 21 groups (n = 397) were assigned to the control condition and used a more traditional after school curriculum focusing on holidays and games. Data were collected from treatment and control children at baseline and after each of three subsequent program implementation years. The Revised Protective Factors Index (Springer & Phillips, 1997), administered in small groups, was used to assess students' capacities on a wide range of social and interpersonal skills, including self-control and confidence. Analyses controlled for gender, program attendance, age, and years in the program, and the study authors found no evidence that attrition from the study differed significantly between the treatment and control conditions. In the final year of the program, treatment group students made greater gains in many of the capacities measured by the RPFI, including self-control.

Major (2013) examined the effect of a physical fitness program, the Friend Fitness program, on grit. The intervention is a school-based mentoring intervention program that challenges at-risk students to give maximum effort through intense physical fitness exercises. The purpose is to teach students to set meaningful goals and then teach them the skills needed to achieve those goals. Relationships with mentors help adolescents learn to overcome formidable challenges and become successful today and in their future. If successful, students are given the opportunity to become mentors to younger students. Engagement in the program teaches children to forge character traits such as grit through their effort. Grit is defined as passion and perseverance for the achievement of long-term goals. The study looked at a sample of 33 adolescents that completed the pre- and mid-program surveys within their initial six months with the program. As part of the survey, participants completed the short grit (Grit-S) scale (Duckworth & Quinn, 2009) as a measure of perseverance of effort. The researchers found a medium effect size of .30 when comparing student responses to the Grit-S at pre-test to mid-program assessment which provides preliminary support for the idea that participation in a mentoring program can increase levels of grit.

Additionally, Yeager et al. (2014) found that perseverance is malleable. Yeager et al. (2014) conducted multiple studies to look at the relationships between purpose for learning and perseverance, self-regulatory behavior, and academic achievement. In a pilot study, the authors evaluated the ability of an intervention focused on self-transcendent purpose to promote personal meaning in school. A total of 451 high school students from 13 different high schools were randomly assigned to the experimental or control groups. The experimental group completed an intervention in which students wrote an open-ended essay related to social injustices, completed structured reading and writing exercises, reviewed results of surveys that promoted prosocial ends, and wrote testimonials to future students regarding prosocial concerns. Students in the control group completed a neutral activity. Both groups of students completed a questionnaire that included measures of (1) self-transcendent, self-oriented, extrinsic motives; (2) persistence (3) self-control; and (4) meaningfulness of their schoolwork. Students in the treatment group reported more personal meaningfulness of tedious academic tasks when compared to students in the control group ($t[446]=2.67$, $p=.007$, $d=.25$). In a follow-up study the authors looked at the effects of the intervention on students' attempts to seriously review academic material. A total of 89 undergraduate students were randomly assigned to the treatment or control group. Students in the treatment group completed a similar intervention to the one described previously but did so online; students in the control group also completed a neutral online activity. Following the online intervention, students completed a tedious review activity that required them to answer over 100 multiple-choice items and the amount of time devoted to each item was measured. Students who completed the intervention showed more perseverance in this activity as evidenced by the greater average time devoted to each question when compared to the that of the control group ($t(69)=2.11$, $p=0.38$, $d=.56$).

In another study, Yeager et al. (2014) examined whether the intervention could lead students to self-regulate. A total of 429 undergraduate students participated in the study for course credit. Participating students were randomly assigned to one of three groups – two treatment groups and a control group. Students who completed the intervention described above completed 36% more boring mathematics problems compared to students in the control group, which was a significant difference. The authors interpret these findings as evidence that purpose for learning is a prominent precursor and mitigator of self-regulatory behavior and perseverance.

The Algebra Project provides extended learning opportunities for students through the use of double mathematics periods throughout high school, the opportunity to participate in summer workshops, and additional support for student learning through the Young Peoples Project (YPP). YPP recruits and trains Math Literacy Workers (MLWs) from the high school to serve as mentors to elementary school students and supplement students' formal educational experiences with high-quality informal educational opportunities outside of the regular school day.

Claim 10

Students who display positive academic behaviors and mindset may develop a greater understanding of mathematics concepts.

Research has shown a positive relationship between positive academic behaviors (e.g., the amount of responsibility students take for their own learning, the amount of responsibility they take for the learning of peers, student beliefs about the discipline of mathematics, and student beliefs about themselves as mathematicians) and the development of mathematical knowledge and skills.

Normative declines in self-esteem, school engagement, and grades are typical in the early adolescent years and are correlated with students' mindsets with respect to intelligence as shown by Blackwell, Trzesniewski, and Dweck (2007). The authors conducted a second, related study to explore the impact of an intervention targeting students' mindset with respect to intelligence on mathematics achievement during middle school. They tested their hypothesis that teaching students to think of their intelligence as a malleable rather than fixed entity would increase their motivation in the classroom and therefore increase achievement. The study was conducted with 99 grade seven students in a public secondary school in New York City. The participants were 52% African American, 45% Latino, and 3% White and Asian and were relatively low-achieving. A total of 91 students were included in the analysis with 48 students in the experimental group and 43 students in the control group. The results showed that both groups experienced an initial decline in grades and that the downward trajectory was halted for the experimental group within a few months of the intervention. The effect of the experimental treatment was significant ($b = 0.53$, $t = 2.93$, $p < 0.05$). Additionally, positive changes in classroom motivation were observed for students in the experimental group ($X^2 = 4.72$, odds ratio = 3.26, $p < .05$).

Oyserman, Bybee, and Terry (2006) base their work on prior evidence linking the difficulty low-income and minority students have envisioning an academic possible self (APS) and integrating that with their social identities, which makes difficult the self-regulatory behavior required to achieve academically. The authors developed an intervention intended to evoke future or possible selves, provide strategies to attain them, and create a link between social identity and possible selves to test the hypotheses that possible selves can be influenced and that academic outcomes are mediated by the intervention's effect on possible selves. The authors conducted a randomized control trial at three Detroit middle schools with 228 students (intervention n = 116; control n = 112), the majority of whom were African American (71.6%) and predominantly low-income as evidenced by the rate of participation in the school lunch program and SES data of the schools' sending communities. Students in the intervention group showed significantly better grade point average ($B = 0.22$, $p < .05$, standardized effect size = 0.25) and standardized test scores ($B = 0.09$, $p < 0.01$, standardized effect size = 0.36) as measured at the end of the academic year of the intervention. Furthermore, the gap between the intervention and control groups increased by the end of Year 2 of the longitudinal study (p. 197) with no additional intervention.

The Algebra Project focuses on establishing a classroom culture that supports student learning by providing resources teachers to set high expectations for positive academic behaviors (including attendance and participation) and by supporting teachers in their use of group participation, and encouraging students to display and share work. Students are continually encouraged to think of mathematics as something accessible and attainable.

Claim 11

Students who persevere in their efforts to learn and do mathematics may develop a greater understanding of mathematics.

The ability to persevere with learning, meaning continuing to pursue knowledge and skills despite difficulties is an essential skill in order to deeply engage with the content. While we were unable to locate literature related to the effects of perseverance on mathematics understanding, a related concept (i.e., self-control) has been linked to improvements in students' mathematical understanding.

Bertrams (2012) investigated the role of the self-control aspect of conscientiousness in mathematics achievement and specifically investigated whether self-control capacity moderated the relationship between students' minimal mathematics grade goals and their actual mathematics grades. The study sample included 172 German ninth-grade students attending a vocational high school. A self-report questionnaire was used as a measure of self-control capacity. Students reported the minimum grade they wanted to achieve on the next written mathematics test. Students' actual mathematics test grade was used as an indicator of mathematics achievement. A hierarchical multiple regression analysis of mathematics test grade was performed to determine the interaction between minimal grade goal and self-control capacity, while controlling for self-efficacy and test anxiety. The prediction of students' mathematics test grade by minimal grade goal varied with the degree of self-control capacity. In other words, students' minimal mathematics grade goals more strongly predicted the actual mathematics grades when students reported higher levels of self-control capacity.

Duckworth and Seligman (2006) investigated the role of gender in the relationship between self-control and mathematics achievement. Their sample was high-achieving, college-bound 8th graders. They used a battery of self-control measures, including self-report questionnaires, teacher and parent questionnaires, and delay of gratification questionnaires. These measures converged and were used to create a composite self-control score. School records including mathematics course level and final report card grades were used as a measure of achievement. This study found a significant gender difference in both students' grades in Algebra I ($n = 111$, $d=.60$, $p<.01$) and students' overall grade point average ($n = 140$, $d=.66$, $p<.001$) with girls outperforming boys. A gender gap in self-control was also shown, with girls rated higher than boys in self-control on all measures. As a result of these differences, over the course of a year, girls spent roughly twice as much time on homework as boys. This study found that composite self-control mediates the relationship between gender and overall grade point average ($\beta = .33$, $p < .001$) indicating that self-control is an important variable when considering mathematics understanding.

The Algebra Project. The summer workshops and professional development for the teachers provide strategies for teachers to use throughout the school year and across courses to encourage students to persevere in their efforts with mathematics. The cohort model for students is also intended to support students to develop close working relationships with a small group of students in order to build trust and to support each other's struggles.

Claim 12

Students who develop positive academic behaviors and mindset may become agents for change for themselves and others

Agency is the belief in one's ability to succeed in specific situations, and it may stem in part from students first developing a set of positive academic behaviors and attitudes about their ability to learn. Positive academic behaviors can be defined as the amount of responsibility students take for their own learning, the amount of responsibility they take for the learning of peers, student beliefs about the discipline of mathematics, and student beliefs about themselves as mathematicians. While the algebra project assumes a causal relationship, only correlational evidence could be located.

Pintrich and De Groot (1990) investigated the relationship between cognitive engagement and motivational orientation by conducting a correlational study for the following components of these constructs: self-efficacy, intrinsic value, self-regulation, and cognitive strategy use. Self-efficacy is closely related to agency, intrinsic value is closely related to academic mindset, and self-regulation and cognitive strategy use together are closely related to academic behavior. A total of 173 middleclass, seventh-grade students completed a self-report questionnaire that was created from items adapted from various existing instruments. Factor analysis was used to guide scale construction for the component constructs: self-efficacy (α=.89), intrinsic value (α=.87), self-regulation (α=.74), and cognitive strategy use (α=.83). The results of a correlation analysis revealed that self-efficacy was correlated with higher levels of intrinsic value (r=.48, p<.001), cognitive strategy use (r=.33, p<.001) and self-regulation (r=.44, p<.001). These results therefore indicate a positive correlation between agency and both academic mindset and academic behavior.

The Algebra Project encourages positive academic behaviors and mindset throughout the program. Recognizing that the students who most need the support of the Algebra Project have likely had multiple years of failure in mathematics, the curriculum is designed to be accessible by connecting hands-on experiences to mathematical concepts in order to help students have positive learning experiences and to build on those experiences to develop longer term behaviors and mindset.

Claim 13

Students who develop cognitive and non-cognitive skills aligned with the theory of action for the Algebra Project have an increased likelihood of graduating from high school in four years college and career ready.

Graduating from high school on time and with the knowledge and skills necessary for college or career requires students to develop an integrated set of cognitive and non-cognitive skills. Career readiness is defined as proficiency in core academic subjects and technical topics, combined with an individual's understanding of professional interests, talents and weaknesses and a solid grasp of the skills and dispositions necessary for engaging in today's fast-paced, global economy. Career readiness may include the non-cognitive skills of communication, critical thinking and problem solving, and collaboration, in addition to goal setting and organizational skills and facility with technology (Career Readiness Partner Council, 2012).While research on the variables that lead to students graduating on time and being prepared for college and careers is limited, there is research on the relationship between the development of cognitive and non cognitive skills and achievement in school.

Blackwell et al. (2007) conducted two studies exploring relationships between beliefs about intelligence and adolescents' mathematics achievement. In the first study the authors followed 373 7th grade students across the transition to 7th grade to assess the extent to which the belief that intelligence is malleable (incremental theory) influenced students' mathematics grades. The study assessed the extent to which students believed math intelligence was malleable, along with other variables related to motivation, at the beginning of 7th grade and then monitored their math grades over the subsequent two years. Although students with fixed and growth mindsets had entered 7th grade with equal prior math achievement, by the end of the 7th grade fall term students with incremental beliefs in math intelligence had begun to outperform those with fixed beliefs regarding math intelligence, and the gap between the two groups only widened over time.

As students made the transition to junior high school, their theory of intelligence became a significant predictor of their mathematics achievement. Moreover, an incremental theory of intelligence at the beginning of junior high school predicted higher mathematics grades earned at the end of the second year of junior high school (β=.17, t=3.40, p<.05), controlling for the effect of math achievement test scores before entering junior high school (β=.43, t=8.48, p<.05). This result held (β=.10, t=2.50, p<.05) using math grades earned in the first term of junior high school (β=.70, t=17.50, p<.05) instead of end of sixth-grade test scores. The second study focused on the effects of an intervention to teach students to see intelligence as a malleable rather than a fixed construct. The main research question focused on whether teaching students to perceive intelligence as incremental led to higher non-cognitive and cognitive outcomes (i.e. motivation and performance) within a mathematics domain relative to not being taught this theory of intelligence. The sample included 91 seventh graders (mainly Black or Hispanic), of which 48 students were assigned to the treatment condition, and 43 were in the control group. There were no significant differences in prior achievement or motivation between the two groups. Results showed that participants in the experimental group changed their opinions towards intelligence (t=3.57, p<.05, Cohen's d=.66) indicating that the intervention group endorsed the incremental theory more strongly after the intervention. Additionally, the change exhibited by the experimental group was significantly more than that of the control group (F=-3.98, p >.05). In addition, academic achievement was measured by mathematics tests at several points throughout the study. Both the experimental and control group initially showed an overall decrease in academic achievement with less of a decrease for the experimental group. At the second measurement of academic achievement, improvements were observed for the intervention group (b=.53, t=2.93, p<.05).

Mega, Ronconi, and De Beni (2014) conducted a study linking non-cognitive factors to academic achievement. Almost 6,000 undergraduate students (63.6% female) participated in the investigation. Each student completed a set of non-cognitive questionnaires with a particular focus on self regulation and motivation. Motivation was represented by several observable variables such as confidence in one's intelligence and self-efficacy. Self-regulated learning was measured using a 48-item questionnaire designed to assess five latent variables: organization, elaboration, self-evaluation, strategies for studying for an exam, and metacognition. The academic achievement was measured as a proportion of the number of exams passed to the number of years spent at the university multiplied by student's grade point average. Results from a structural equation model, in which self-regulated learning and motivation were modeled as mediators between emotion and academic achievement, showed that both self-regulated learning and motivation positively impacted academic achievement with motivation having a larger effect (β=.16 and β=.32, respectively).

Connell, Spencer, and Aber (1994) examined the validity of a model of human motivation as it applies to school success. A total of 728 6th to 9th grade students participated in the study, which included three samples, one in Atlanta, a second in New York, and a third in Washington, D.C.. The majority of participating students were African American. The authors assessed how contextual factors and self perceptions related to measures of educational outcomes combined into an index of resiliency. Measures of context assessed demographic variables including gender, neighborhood risk, family structure, and single female-headed family. Measures of self included perceived academic competence, perceived self-worth and positive feelings toward self, peer acceptance, and engagement in schooling. Measures of educational outcomes included attendance, performance on state reading and mathematics assessments, grade point averages, suspensions, and student age relative to grade level (indicating retention). Significant and robust correlations were obtained among model variables in all three samples.

Conley, McGaughy, Kirtner, van der Valk, and Martinez-Wenzl (2010) discuss findings from a qualitative study where the goal was to validate and operationalize a definition of college readiness by examining programs and practices across 38 public schools, the majority of which served underrepresented groups. Examining data across 300 interviews/focus groups, 224 classroom observations and 640 documents collected for analysis, authors identified seven key principles for promoting college readiness. This work resulted in the creation of the CollegeCareerReady School Diagnostic instrument, which measures school-level college readiness over time. This paper lends support to an emerging understanding that students who wish to be college ready must develop a set of cognitive strategies—specifically, interpretation, precision/accuracy, problem formulation, reasoning, and research. Authors also assert that students "who do not challenge themselves academically during the senior year are much more likely to place in remedial courses in college and earn lower grades in entry-level college courses. These effects are particularly pronounced for first generation college attendees, low-income students, or racial or ethnic minorities" (p. 22). Finally, authors advocate for high school seniors to register for challenging courses (including math) regardless of the number of credits they need to graduate, participate in intensive seminars that focus on in-depth content knowledge as well as experience internships that keep students academically engaged.

The Algebra Project uses a multi-pronged approach to provide curriculum to make mathematics accessible, to provide students with experiences where they feel successful in order to counter their previous school experiences, and to develop a cohort model for both teacher and student support. The content is not "dumbed down" for students since the goal is that they are college and career ready, but additional supports are provided to make that aspiration possible. To achieve the goal of college and career readiness, the Algebra Project curriculum explicitly focuses on teaching students that math intelligence is malleable, in addition to teaching math skills using an applied and engaging approach, and targeting student motivation by increasing students' enjoyment of mathematics and their understanding of the real-world applicability of mathematics skills and knowledge.

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